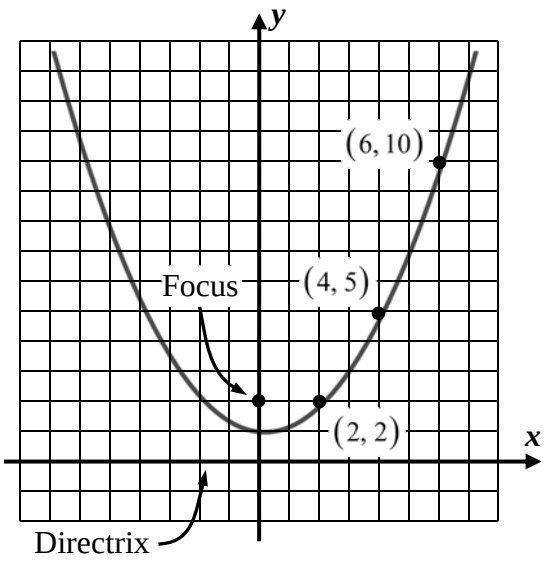
Name: _____ Date: _____

A2CC Locus Definition of a Parabola

THE LOCUS DEFINITION OF A PARABOLA

A parabola is the collection of all points **equidistant** from a fixed point (known as its **focus**) and a fixed line (known as its **directrix**).

***Exercise* #1:** The parabola $y = \frac{1}{4}x^2 + 1$ is shown graphed below with selected points shown. For this parabola, its focus is the point $(0, 2)$ and its directrix is the *x*-axis.



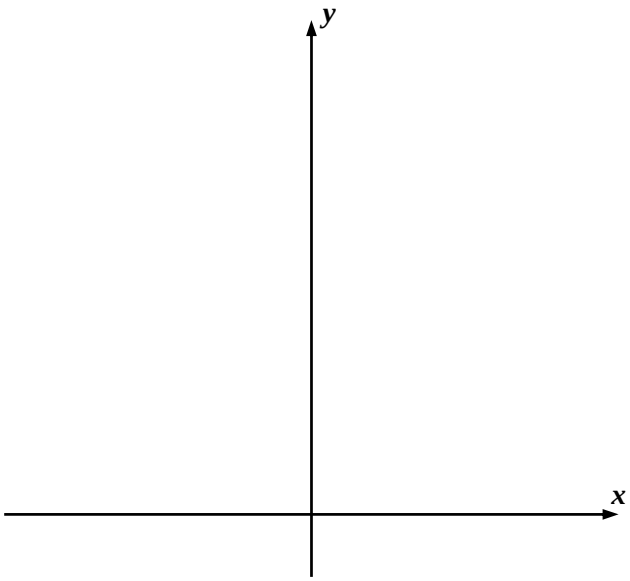
(a) How far is the turning point $(0,1)$ from both the focus and directrix? How far is the point $(2, 2)$ from both?

(b) Use the distance formula to verify that the point $(4, 5)$ is the same distance away from the focus and directrix. Draw line segments from the focus and directrix to this point to visualize the distance. Repeat for the point $(6, 10)$

(c) Use the distance formula to show that the equation of this parabola is $y = \frac{1}{4}x^2 + 1$ based on the locus definition of a parabola.

***Exercise* #2:** Consider a parabola whose focus is the point $(0, 7)$ and whose directrix is the line $y = 3$.

(a) Sketch a diagram of the parabola below and identify its turning point.



(b) Determine the equation of the parabola using the locus definition.

Any line and any point not on the line when used as the focus and directrix define a parabola. The most challenging type of problem we will tackle in this course will be finding the equation of a parabola whose focus point is not on one of the two axes. We will, however, stick with horizontal lines as our directrices.

***Exercise* #3:** Determine the equation of the parabola whose focus is the point $(4, 1)$ and whose directrix is the horizontal line $y = -3$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.

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HOMEWORK

1. Fill in the following locus definition of a parabola with one of the words shown listed below. Words may be used more than once.

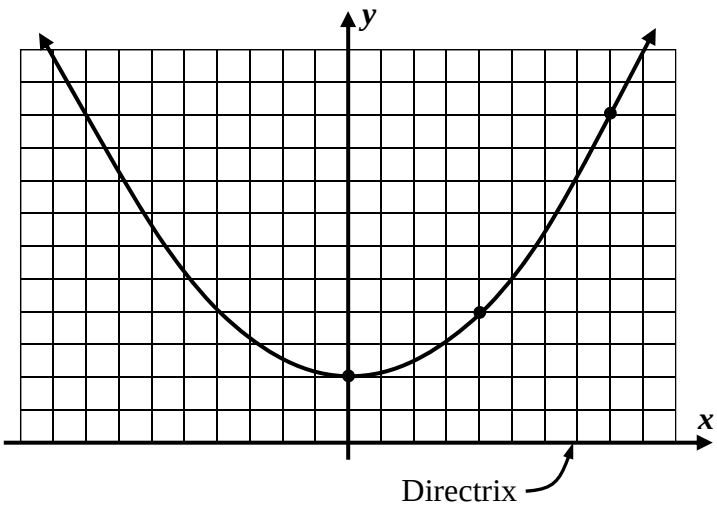
   point, line, equidistant, directrix, collection, focus

   A parabola is the _____ of all points _____ from a fixed _____ and a fixed _____.

   The fixed _____ is known as the parabola's _____.

   The fixed _____ is known as the parabola's _____.

2. The parabola whose equation is $y = \frac{1}{8}x^2 + 2$ is shown graphed on the grid below. Its directrix is the *x*-axis.



   (a) Explain why the focus must be the point $(0, 4)$. Label this point on the diagram.

   (b) How far is the point $(4, 4)$ from both the focus and the directrix?

   (c) Show that the point $(8, 10)$ is equidistant from the focus and directrix.

   (d) Using the locus definition of a parabola, show that the equation is $y = \frac{1}{8}x^2 + 2$.

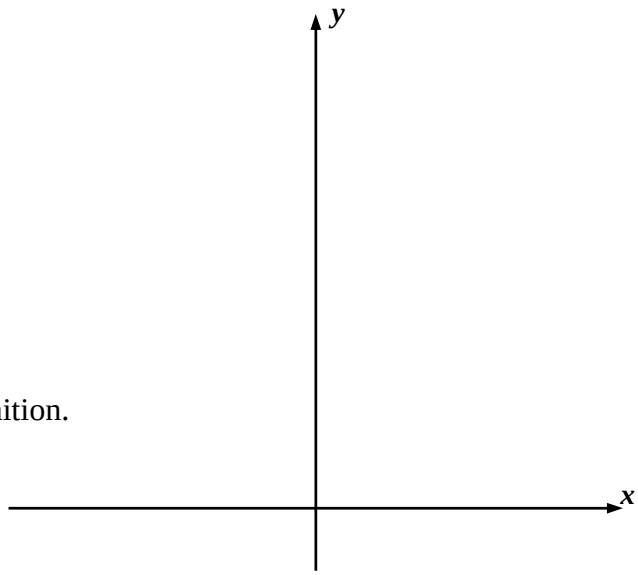
3. Consider parabola that is the collection of all points equidistant from the point $(0, 8)$ and the line $y = 2$.

(a) Give each of the following:

Directrix: _____

Focus: _____

(b) Draw a diagram of this parabola and label its turning point on the diagram below.



(c) Find the equation of this parabola using the locus definition.

4. Parabolas can be constructed using the classic geometric tools of a compass and a straightedge. The circles below represent all the points equidistant from the focus $(0, 4)$. Given this focus point and a directrix of the *x*-axis, do the following.

(a) Draw in the horizontal lines $y = 2$, $y = 3$, $y = 4$, $y = 5$, $y = 6$, $y = 7$, and $y = 8$. These lines represent points that are a fixed distance away from the *x*-axis (the directrix).

(b) Plot points at the intersections of the lines your drew in (a) with the circles that are the same distance from the focus. Connect with a smooth parabolic curve.